Bounds for Optimal Golomb Rulers

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Abstract

In this paper, we discuss some bounds for optimal Golomb rulers.

Definition

A Golomb ruler, named after Solomon W. Golomb and discovered independently by others, is a set of marks at integer positions along a ruler such that no two pairs of marks are the same distance apart.[1] The number of marks on the ruler is its order and the length is the maximum distance between two of its marks. It is customary to use 0 as the first mark and n as the last.

Flipping the ruler allows us to generate twice the number of rulers with different starting points.

For example, [0, 1, 3, 7] is a Golomb ruler consisting of four marks where the four marks' consecutive distances are 1, 2, and 4. Subtracting the ruler from the maximum number (i.e., 7) and taking its reflection gives us [0, 4, 6, 7]. These two rulers are considered equal since the spacings are the same in reverse order: (1, 2, 4) for the first and (4, 2, 1) for its reflection. Without the numbers, the two rules are the same when rotated. Hence, every Golomb ruler has a unique, and thus equal, rotation. A symmetrical Golomb ruler would break the rule of the distances being unique.

For order n, there are ${}_{n-1}C_2 = \binom{n-1}{2} = \frac{\mathrm{n(n-1)}}{2}$ distances to compare.

An optimal Golomb ruler (OGR) is the smallest length of a fixed order.

Note: An OGR for a fixed n is not necessarily unique as seen in Table 1.

A perfect Golomb ruler has all ${}_{n-1}C_2$ distances. Only O(1) to O(4) are perfect. All others have been proven not to be.[1]

OGR Analysis

The following table shows the OGRs up to order 28 and their lengths and marks.

Table 1 – OGRs up to O(28)

O(n)	L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0	0																											
2	1	0	1																										
3	3	0	1	3																									
4	6	0	1	4	6																								
5	11	0	1	4	9	11																							
5	11	0	2	7	8	11																							
6	17	0	1	4	10	12	17																						
6	17	0	1	4	10	15	17																						
6	17	0	1	8	11	13	17																						
6	17	0	1	8	12	14	17																						
7	25	0	1	4	10	18	23	25																					
7	25	0	1	7	11	20	23	25																					
7	25	0	1	11	16	19	23	25																					
7	25	0	2	3	10	16	21	25																					
7	25	0	2	7	13	21	22	25																					
8	34	0	1	4	9	15	22	32	34																				
9	44	0	1	5	12	25	27	35	41	44																			
10	55	0	1	6	10	23	26	34	41	53	55																		
11	72	0	1	4	13	28	33	47	54	64	70	72																	
11	72	0	1	9	19	24	31	52	56	58	69	72																	
12	85	0	2	6	24	29	40	43	55	68	75	76	85																
13	106	0	2	5	25	37	43	59	70	85	89	98	99	106															
14	127	0	4	6	20	35	52	59	77	78	86	89	99	122	127														
15	151	0	4	20	30	57	59	62	76	100	111	123	136	144	145	151													
16	177	0	1	4	11	26	32	56	68	76	115	117	134	150	163	168	177												
17	199	0	5	7	17	52	56	67	80	81	100	122	138	159	165	168	191	199											
18	216	0	2	10	22	53	56	82	83	89	98	130	148	153	167	188	192	205	216										
19	246	0	1	6	25	32	72	100	108	120	130	153	169	187	190	204	231	233	242	246									
20	283	0	1	8	11	68	77	94	116	121	156	158	179	194	208	212	228	240	253	259	283								
21	333	0	2	24	56	77	82	83	95	129	144	179	186	195	255	265	285	293	296	310	329	333							
22	356	0	1	9	14	43	70	106	122	124	128	159	179	204	223	253	263	270	291	330	341	353	356						
23	372	0	3	7	17	61	66	91	99	114	159	171	199	200	226	235	246	277	316	329	348	350	366	372					
24	425	0	9	33	37	38	97	122	129	140	142	152	191	205	208	252	278	286	326	332	353	368	384	403	425				
25	480	0	12	29	39	72	91	146	157	160	161	166	191	207	214	258	290	316	354	372	394	396	431	459	467	480			
26	492	0	1	33	83	104	110	124	163	185	200	203	249	251	258	314	318	343	356	386	430	440	456	464	475	487	492		
27	553	0	3	15	41	66	95	97	106	142	152	220	221	225	242	295	330	338	354	382	388	402	415	486	504	523	546	553	
28	585	0	3	15	41	66	95	97	106	142	152	220	221	225	242	295	330	338	354	382	388	402	415	486	504	523	546	553	585

Upper-Bounds

Trivial UB ($_{n-1}C_2$)

In order to create a Golomb ruler, one can use the simplest approach. Starting with a sequence, add one more than the length to ensure that all new delta distances are unique.

Thus, starting with 0, we add 1 to get 1, then 2 to get 3, then 4 to get 7, etc. The numbers 0, 1, 3, 7 are the lengths that are guaranteed to generate a Golomb ruler based on the previous order.

Thus, UB(n) = [0, 1, 3, 7, …, 2^{n-1}-1].

Unfortunately, this geometric series grows rapidly and, thus, makes for a bad upper-bound.

Fast UB (2 * Prev + 1)

A slightly better approach would be to take the previous order's length and add one more than the length to ensure that all new delta distances are unique.

Thus, starting with 0, we add 1 to get 1, then add 2 to get 3, then add 4 to get 7. However, since we know that the O(4) is of length 6, we add 7 to get 13 as the upper-bound for O(5). Since the optimal length of O(5) is 11, we add 12 to get an upper-bound of 23 for O(6). These upper-bounds are guaranteed to generate a Golomb ruler based on the previous order's optimal length.

Smart UB

We can find an even lower upper-bound for the higher orders. The order of a ruler is quite small when compared to the length. Thus, the total number of unused marks ($L - {}_{n-1}C_2$) is quite large.

For each OGR, find the smallest unused number (SUN) that, when added to the OGR length, does not generate any existing delta differences and becomes the new maximum for the next OGR. In other words, the n+1 new deltas that it generates belong to non-Golomb numbers (highlighted in yellow in Table 2). If the previous set is perfect or no SUN exists (potentially possible, but seems unlikely), then add one more than the length. Numbers highlighted in orange are for orders with multiple lengths and the row with the smallest SUN is chosen.

Note:

1. The deltas of the yellows match the deltas of the OGRs in reverse order.
2. The count of numbers not in a Golomb ruler is $L - {}_{n-1}C_2$. Although this count appears to be increasing, it is not guaranteed as can be seen for O(23) and O(26). The reason for this is due to the SUN chosen and the spread of the OGR numbers.
3. Similarly, the UBs for consecutive numbers are not increasing as can be seen for UB(18), UB(20), UB(24), and UB(28).
4. Only new deltas from $L_{n+1} = L_n$ + SUN will have to be tested, and not all n new deltas. $L_{n+1} - L_n$ is safe by the definition of SUN. A fixed number from the start will also not have to be tested since some will potentially be bigger than L_n. From largest to smallest, stop testing at the delta larger than L_n.
5. Interestingly enough, UB(28) is exactly its OGR length.

Since UB(29) is equal to 757. It might be faster to test all 29-mark rulers starting at length 757 and test the lengths down. Obviously, to find the smallest length, we would still need to test all the lower-bounds, but this approach might give us a contender faster.

Table 2 – Upper-Bounds for OGRs up to O(29)

O(n)	$_{n-1}C_2$	L	SUN	UB	Numbers Not in Golomb Ruler
1	0	0	1	0	∅
2	1	1	2	1	∅
3	3	3	4	3	∅
4	6	6	7	7	∅
5	10	11	12	13	[6]
5	10	11	12	13	[10]
6	15	17	14	23	[14, 15]
6	15	17	18	23	[8, 12]
6	15	17	14	23	[14, 15]
6	15	17	18	23	[10, 15]
7	21	25	26	31	[11, 12, 16, 20]
7	21	25	26	31	[8, 15, 17, 21]
7	21	25	26	31	[13, 17, 20, 21]
7	21	25	20	31	[12, 17, 20, 24]
7	21	25	26	31	[10, 16, 17, 24]
8	28	34	24	45	[16, 20, 24, 26, 27, 29]
9	36	44	28	58	[18, 21, 28, 31, 33, 37, 38, 42]
10	45	55	36	72	[36, 37, 38, 39, 42, 44, 46, 48, 50, 51]
11	55	72	65	91	[11, 22, 30, 35, 38, 40, 45, 48, 49, 52, 55, 56, 58, 61, 62, 65, 67]
11	55	72	26	91	[26, 29, 35, 36, 40, 42, 44, 46, 54, 59, 61, 62, 64, 65, 66, 67, 70]
12	66	85	48	98	[48, 50, 54, 57, 58, 59, 60, 63, 64, 65, 67, 71, 72, 77, 78, 80, 81, 82, 84]
13	78	106	71	133	[24, 31, 44, 49, 50, 51, 53, 58, 66, 67, 71, 72, 75, 76, 77, 78, 79, 82, 86, 88, 90, 91, 92, 95, 100, 102, 103, 105]
14	91	127	56	177	[56, 60, 61, 62, 65, 67, 76, 81, 84, 88, 90, 91, 94, 96, 97, 98, 100, 101, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 117, 119, 120, 124, 125, 126]
15	105	151	98	183	[18, 31, 48, 50, 63, 65, 67, 71, 73, 78, 84, 90, 95, 97, 98, 99, 101, 102, 104, 105, 108, 109, 110, 112, 113, 117, 118, 120, 122, 126, 127, 128, 129, 130, 133, 134, 135, 137, 138, 139, 142, 143, 146, 148, 149, 150]
16	120	177	126	249	[23, 37, 38, 40, 54, 63, 69, 70, 71, 73, 77, 79, 80, 81, 84, 86, 88, 90, 93, 96, 97, 98, 99, 103, 105, 110, 119, 120, 122, 125, 126, 127, 128, 129, 132, 135, 138, 140, 141, 143, 144, 147, 148, 153, 154, 155, 156, 158, 160, 161, 165, 169, 170, 171, 172, 174, 175]
17	136	199	89	303	[18, 36, 54, 72, 89, 90, 94, 96, 97, 102, 104, 106, 108, 114, 120, 123, 125, 126, 127, 128, 129, 130, 134, 136, 137, 140, 141, 144, 145, 146, 149, 150, 153, 155, 156, 157, 162, 164, 166, 167, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 183, 185, 187, 188, 189, 190, 193, 195, 196, 197, 198]
18	153	216	91	288	[91, 93, 101, 102, 104, 112, 113, 115, 117, 119, 121, 124, 125, 129, 137, 140, 141, 142, 144, 147, 150, 154, 155, 156, 158, 159, 161, 162, 164, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 184, 185, 187, 189, 191, 193, 196, 197, 198, 199, 200, 201, 202, 204, 207, 208, 209, 210, 211, 212, 213, 215]
19	171	246	50	307	[50, 54, 63, 65, 85, 86, 91, 92, 106, 109, 110, 117, 127, 135, 136, 139, 140, 141, 143, 145, 148, 149, 150, 151, 154, 156, 157, 160, 164, 166, 167, 171, 173, 175, 176, 177, 178, 180, 182, 183, 185, 188, 191, 192, 193, 194, 195, 196, 197, 200, 202, 205, 207, 209, 211, 212, 213, 215, 216, 218, 219, 220, 222, 223, 224, 226, 228, 229, 234, 235, 237, 238, 239, 243, 244]
20	190	283	98	296	[98, 99, 106, 109, 122, 123, 128, 129, 130, 133, 136, 139, 141, 142, 149, 152, 153, 154, 161, 164, 166, 169, 170, 173, 174, 175, 177, 180, 181, 184, 187, 188, 190, 192, 195, 196, 198, 199, 202, 203, 205, 209, 210, 213, 214, 216, 218, 219, 221, 222, 223, 224, 225, 226, 230, 231, 233, 234, 235, 236, 237, 238, 241, 243, 244, 246, 247, 249, 250, 254, 255, 256, 257, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 276, 277, 278, 279, 280, 281]
21	210	333	85	381	[29, 43, 63, 65, 72, 85, 87, 89, 92, 94, 108, 116, 119, 122, 125, 128, 132, 133, 135, 137, 140, 145, 146, 148, 151, 153, 157, 158, 159, 161, 163, 165, 168, 169, 174, 175, 176, 180, 187, 191, 192, 194, 196, 197, 205, 206, 207, 212, 217, 218, 220, 221, 222, 223, 224, 225, 226, 230, 232, 235, 236, 239, 242, 243, 244, 245, 248, 249, 257, 258, 259, 260, 262, 264, 266, 267, 268, 270, 271, 274, 275, 276, 278, 279, 280, 281, 282, 284, 287, 288, 289, 290, 292, 295, 297, 298, 299, 300, 301, 302, 303, 304, 306, 307, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 330, 332]
22	231	356	140	418	[24, 32, 33, 41, 46, 48, 72, 75, 96, 102, 120, 138, 140, 143, 144, 154, 155, 156, 160, 166, 168, 172, 173, 175, 176, 181, 184, 186, 187, 188, 189, 191, 192, 196, 198, 199, 201, 205, 207, 211, 212, 215, 216, 218, 226, 230, 233, 236, 237, 238, 240, 241, 242, 243, 245, 246, 251, 255, 257, 258, 259, 264, 265, 266, 267, 268, 272, 273, 274, 275, 276, 278, 279, 280, 281, 284, 285, 288, 289, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 314, 315, 317, 318, 319, 320, 322, 323, 324, 325, 326, 328, 331, 333, 334, 335, 336, 337, 338, 343, 345, 346, 348, 349, 350, 351, 354]
23	253	372	119	496	[62, 65, 69, 79, 119, 123, 125, 128, 141, 143, 153, 161, 162, 175, 176, 181, 184, 187, 188, 190, 194, 198, 203, 204, 205, 206, 208, 210, 212, 214, 220, 221, 222, 224, 227, 231, 233, 237, 240, 241, 242, 244, 245, 247, 248, 253, 254, 256, 261, 262, 264, 265, 266, 269, 271, 272, 276, 278, 279, 280, 283, 285, 286, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 307, 308, 310, 314, 315, 317, 318, 319, 320, 321, 323, 324, 325, 327, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 342, 344, 346, 351, 352, 353, 354, 356, 357, 358, 360, 361, 362, 364, 367, 368, 370, 371]
24	276	425	128	491	[128, 137, 150, 159, 161, 165, 166, 169, 178, 183, 185, 187, 188, 194, 200, 202, 206, 207, 209, 218, 221, 222, 223, 225, 227, 230, 233, 236, 237, 238, 247, 250, 254, 257, 258, 259, 260, 264, 265, 266, 267, 268, 270, 272, 275, 276, 279, 280, 282, 284, 290, 291, 292, 297, 298, 300, 301, 302, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 321, 322, 324, 325, 327, 329, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 345, 348, 349, 350, 352, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 367, 369, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424]
25	300	480	90	553	[81, 90, 93, 103, 110, 111, 120, 139, 153, 171, 172, 174, 176, 183, 184, 192, 196, 198, 200, 204, 210, 213, 216, 220, 221, 223, 227, 231, 232, 238, 241, 242, 243, 247, 249, 254, 255, 256, 257, 259, 262, 264, 267, 269, 272, 275, 279, 280, 283, 284, 286, 288, 291, 292, 294, 295, 296, 297, 308, 309, 311, 312, 317, 318, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 356, 358, 361, 362, 363, 364, 366, 369, 370, 371, 373, 374, 375, 377, 378, 379, 380, 381, 383, 385, 386, 388, 390, 391, 393, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 421, 422, 423, 424, 425, 426, 427, 429, 432, 433, 434, 435, 436, 437, 439, 440, 442, 443, 444, 445, 446, 448, 449, 450, 452, 453, 454, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479]
26	325	492	159	570	[159, 160, 164, 165, 176, 177, 187, 188, 192, 195, 196, 197, 209, 211, 212, 220, 221, 222, 228, 242, 244, 247, 254, 259, 263, 265, 266, 268, 269, 270, 274, 278, 280, 283, 286, 288, 291, 294, 295, 296, 297, 298, 299, 300, 304, 305, 308, 309, 311, 315, 319, 321, 322, 325, 327, 328, 331, 333, 334, 335, 337, 338, 339, 341, 344, 345, 348, 349, 350, 358, 359, 361, 362, 364, 366, 367, 369, 370, 372, 374, 375, 376, 378, 379, 380, 384, 387, 389, 390, 391, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 405, 406, 408, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 432, 433, 434, 435, 436, 437, 438, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 457, 458, 460, 461, 462, 465, 466, 467, 468, 469, 470, 471, 472, 473, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490]
27	351	553	32	651	[32, 39, 62, 81, 99, 170, 172, 175, 183, 187, 197, 203, 204, 207, 211, 213, 214, 219, 226, 231, 234, 237, 238, 245, 247, 249, 252, 253, 255, 256, 267, 268, 269, 270, 271, 274, 275, 277, 278, 286, 290, 294, 299, 300, 301, 306, 308, 310, 312, 314, 317, 319, 324, 329, 331, 337, 340, 342, 343, 345, 346, 348, 350, 353, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 368, 369, 370, 372, 375, 376, 377, 378, 383, 384, 386, 390, 392, 393, 395, 396, 397, 403, 405, 406, 408, 410, 413, 414, 416, 418, 419, 421, 422, 423, 424, 425, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 442, 443, 444, 446, 448, 450, 452, 453, 454, 455, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 477, 478, 479, 481, 484, 485, 488, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 503, 506, 507, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 521, 522, 524, 525, 526, 527, 528, 529, 530, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 544, 545, 547, 548, 549, 551, 552]

O(n)	$_{n-1}C_2$	L	SUN	UB	Numbers Not in Golomb Ruler
28	378	585	172	585	[172, 175, 187, 204, 207, 211, 213, 214, 219, 226, 234, 237, 238, 245, 249, 252, 253, 256, 267, 268, 269, 270, 271, 274, 275, 277, 278, 286, 294, 299, 300, 301, 306, 308, 310, 312, 314, 317, 319, 324, 329, 331, 337, 340, 342, 345, 346, 348, 350, 353, 355, 356, 357, 358, 359, 363, 366, 368, 369, 370, 372, 375, 376, 377, 378, 383, 384, 386, 390, 392, 393, 395, 396, 397, 403, 405, 406, 408, 410, 413, 414, 416, 418, 419, 421, 422, 423, 424, 425, 427, 429, 430, 431, 432, 434, 435, 436, 437, 439, 441, 442, 444, 446, 448, 450, 452, 453, 454, 455, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 477, 478, 481, 484, 485, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 503, 506, 507, 509, 510, 511, 513, 514, 515, 516, 517, 518, 521, 522, 524, 525, 526, 527, 528, 529, 530, 532, 533, 534, 535, 536, 537, 539, 540, 541, 542, 545, 547, 548, 549, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584]

First and Second-to-Last Non-Zero Markers

Now that we have an upper-bound for O(n), we need to also have an upper-bound for the first non-zero marker (L_1) and a lower-bound for the second-to-last non-zero marker (L_{n-1}).

We know that the minimum length of O(n) is $_{n-1}C_2$. Summing forward or backwards, the maximum distance between L_0 (= 0) and L_1, and similarly for L_{n-1} and L_n, is UB − $_{n-2}C_2$. Thus, the maximum L_1 is UB − $_{n-2}C_2$ and the minimum L_{n-1} is $_{n-2}C_2$.

Example:

For n = 7, the upper-bound is 31. Thus, the markers could be anything in between [0, 1, 3, 6, 10, 15, 21] (increasing deltas) or [0, 16, 21, 25, 28, 30, 31] (decreasing deltas) with a length from 21 to 31. L(7) = 25 in this case.

Lower-Bounds

Trivial LB ($_{n-1}C_2$)

For a ruler with n marks, there have to be $_{n-1}C_2$ unique distances between them. Thus, the simplest OGR must be greater than or equal to $_{n-1}C_2$.

Smart LB

Since the length of every OGR of order O(n) must be strictly greater than the length of O(n−1), a better lower-bound is the maximum between $_{n-1}C_2$ and one more than the previous length (L_{n-1} + 1). For n ≤ 10, $_{n-1}C_2$ is the minimum starting point (with equality at 10). However, for n > 10, L_{n-1} + 1 grows more rapidly. The only way for $_{n-1}C_2$ to grow faster is if there are many short length differences (ΔL) among the OGRs.

Table 3 – Lower-Bounds for OGRs up to O(29)

O(n)	$_{n-1}C_2$	L	LB	ΔL
1	0	0	0	–
2	1	1	1	1
3	3	3	3	2
4	6	6	6	3
5	10	11	10	5
6	15	17	15	6
7	21	25	21	8
8	28	34	28	9
9	36	44	36	10
10	45	55	45	11
11	55	72	56	17
12	66	85	73	13
13	78	106	86	21
14	91	127	107	21
15	105	151	128	24

O(n)	$_{n-1}C_2$	L	LB	ΔL
16	120	177	152	26
17	136	199	178	22
18	153	216	200	17
19	171	246	217	30
20	190	283	247	37
21	210	333	284	50
22	231	356	334	23
23	253	372	357	16
24	276	425	373	53
25	300	480	426	55
26	325	492	481	12
27	351	553	493	61
28	378	585	554	32

Other LBs

It is only fair to mention that there are other lower-bound estimates[2][3] for the OGR lengths that don't depend on calculating the previous OGR first.

Optimal Direction

Looking at the lower- and upper-bounds compared to the known lengths, for this approach only, starting from the lower-bound appears to be the better approach almost always.

Table 4 – Bound Distances to Length

O(n)	L - LB	UB - L	Closer
1	0	0	–
2	0	0	–
3	0	0	–
4	0	1	Min
5	1	2	Min
6	2	6	Min
7	4	6	Min
8	6	11	Min
9	8	14	Min
10	10	17	Min
11	16	19	Min
12	12	13	Min
13	20	27	Min
14	20	50	Min
15	23	32	Min
16	25	72	Min
17	21	104	Min
18	16	72	Min
19	29	61	Min
20	36	13	Max
21	49	48	Max
22	22	62	Min
23	15	124	Min
24	52	66	Min
25	54	73	Min
26	11	78	Min
27	60	98	Min
28	31	0	Max

References

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